Notes on MCMC

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1 MCMC

- goal of MCMC: sample from the target distribution $p(x) \propto f(x)$
- rejection sampling: for the target distribution $p(x) \propto f(x)$, sample from $Mg(x)$ where $f(x) \leq Mg(x)$ and $u \sim U[0,1]$, accept if $f(x)/Mg(x) > u$.
- the problem: at x_0 where $f(x_0)/Mg(x_0)$ is high, it is likely that in the neighborhood of x_0 the density of $p(x)$ is high too, but the rejection sampling does not use this information
- high-level idea of MCMC: the next draw depends on the last, and the stationary distribution of the Markov Chain is the target distribution $p(x)$
- Metropolis-Hasting:
  - sample from the easier $g(x_{t+1}|x_t)$ (the proposal distribution, e.g., $g(x_{t+1}|x_t) = N(x_t, \sigma^2)$)
  - detailed balance in the steady state

$$\begin{aligned} f(a)g(b|a)A(a \to b) &= f(b)g(a|b)A(b \to a) \\ \Rightarrow \frac{A(a \to b)}{A(b \to a)} &= \frac{f(b)}{f(a)} \frac{g(a|b)}{g(b|a)} \end{aligned}$$

  - acceptance probability $A(a \to b) = \min\{1, \frac{f(b)}{f(a)} \frac{g(a|b)}{g(b|a)}\}$
  - if $g(x_{t+1}|x_t)$ is symmetric, we have $g(a|b) = g(b|a)$, so the acceptance probability is $\min\{1, p(b)/p(a)\}$, i.e., if $p(b) > p(a)$, move from a to b w.p. 1.
- Gibbs Sampling:
  - sample from a multi-variate distribution $p(x, y)$ where sampling from $p(x|y)$ and $p(y|x)$ is easy
  - sample from the conditional distributions and alternate